Definitely Not Flawed or Not Definitely Flawed?

FPL Away Days 2013

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How to Design a Type System? (1)

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  "the program is definitely not flawed"

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- This underpins notions such as progress and preservation; it would be strange if a flawless program turns out to be flawed once it is run:

  “Well-typed programs do not go wrong [and should therefore be accepted]”

- Of course, depending, we may have to settle for a quite weak notion of “flawless”.

How to Design a Type System? (2)

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- Assume main design goal is to catch as many problems as possible as early as possible.
- Then what about opting for

  "the program is not definitely flawed"

  if that allows a ***stronger*** notion of "flawed", meaning ***more*** problems can be caught?
- Of course, a not definitely flawed program may be revealed to be flawed; e.g. when run.

How to Design a Type System? (3)

- One may question if we then are entitled to talk about a “type system”; Pierece (2002):

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  A type system is a tractable syntactic method for proving the absence of certain [undesirable] program behaviors by classifying phrases according to the kinds of values they compute.

- What we are considering here is proving the ***presence*** of undesirable program behaviours, and rule out the program if so.

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  but do not insist that well-typed programs do not reveal further errors when run.
- A kind of mixed static and dynamic approach to typing.
- Possible to view as instance of other “mixed approaches”, such as Hybrid Type Checking (Flanagan) or Gradual Typing (Siek and Taha)?

This Talk

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- Overview of the equation-system aspect of a type system for Functional Hybrid Modelling (FHM) as an example of opting for “not definitely flawed”.
- Possible points for discussion:
  - Pros and cons of the approach.
  - What meta-theoretical properties can or should one prove?
  - Other instances of (aspects of) type systems like this?

The FHM Type System

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- ***Refinement*** of a standard type system:
  - Progress and preservation holds for the ***unrefined*** system.
- The refinements impose ***additional*** constraints related to “well-formedness” of modular systems of equations:
  - The refined system rules out ***strictly more*** programs than the unrefined one.
  - In particular, definitely ill-formed equation system fragments are rejected.
  - But there is ***no guarantee*** that the resulting system of equations will be well-formed.

The FHM Setting (1)

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- Undirected equations: ***non-causal modelling***. (Differential Algebraic Equations, DAE)
- Two-level design:
  - ***equation level*** for modelling components
  - ***functional level*** for spatial and temporal composition of components

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- Spatial composition: signal relation ***application***; enables modular, hierarchical, system description.
- Temporal composition: ***switching*** from one structural configuration into another.

The FHM Setting (3)

$$\begin{aligned}
&resistor :: \texttt{Resistance} \rightarrow \mathsf{SR}\,(\texttt{Pin}, \texttt{Pin})\\
&resistor\ r = \textbf{sigrel}\ (p, n)\ \textbf{where}\\
&\qquad twoPin \diamond (p, n, u)\\
&\qquad r \cdot p.i = u
\end{aligned}$$

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Parametrised model represented by ***function*** mapping parameters to a model. Note: first class models!

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Encapsulated equation system fragment.

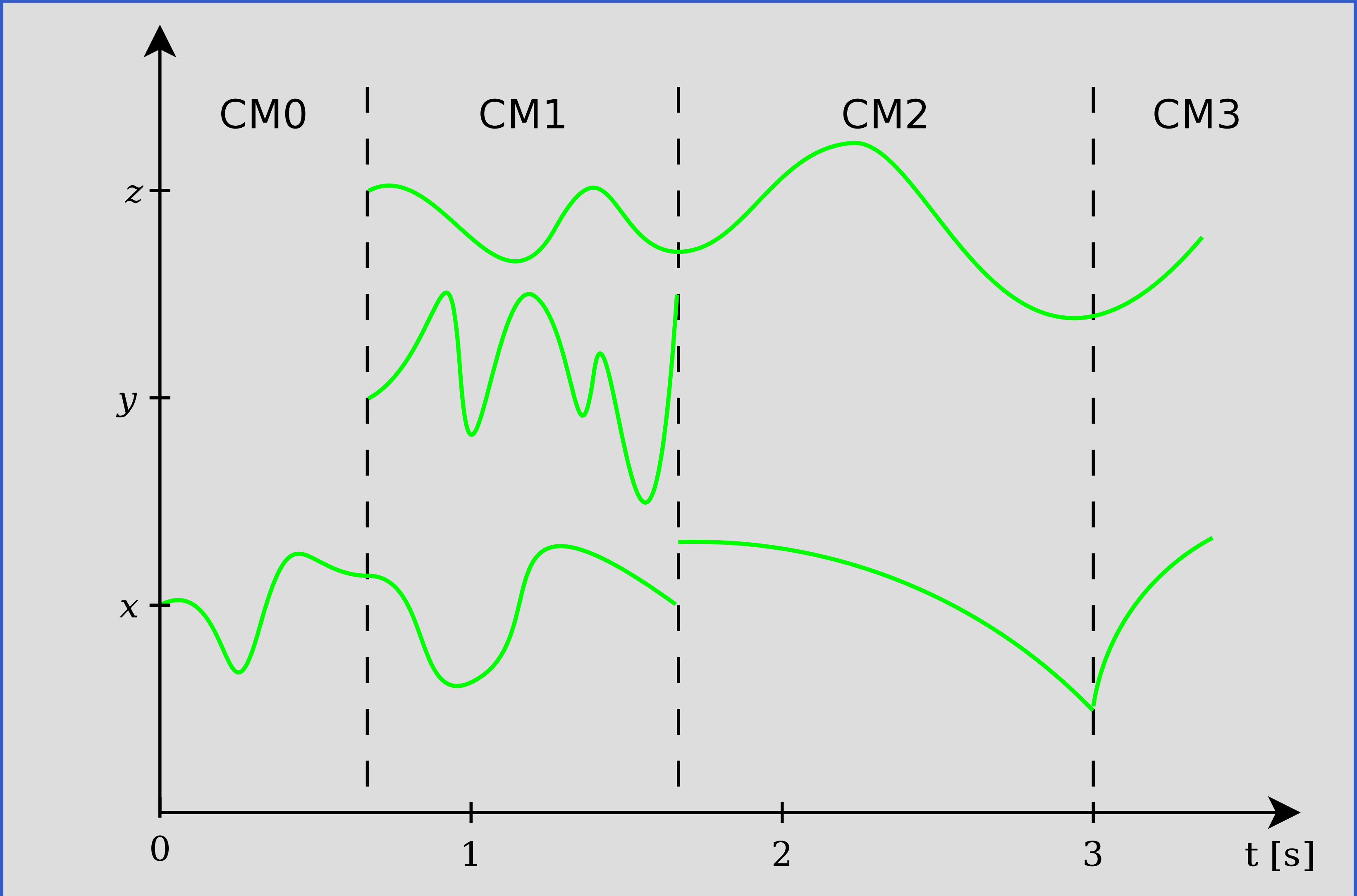
Signal relation ***application*** allows modular construction of models from component models.

The FHM Setting (4)

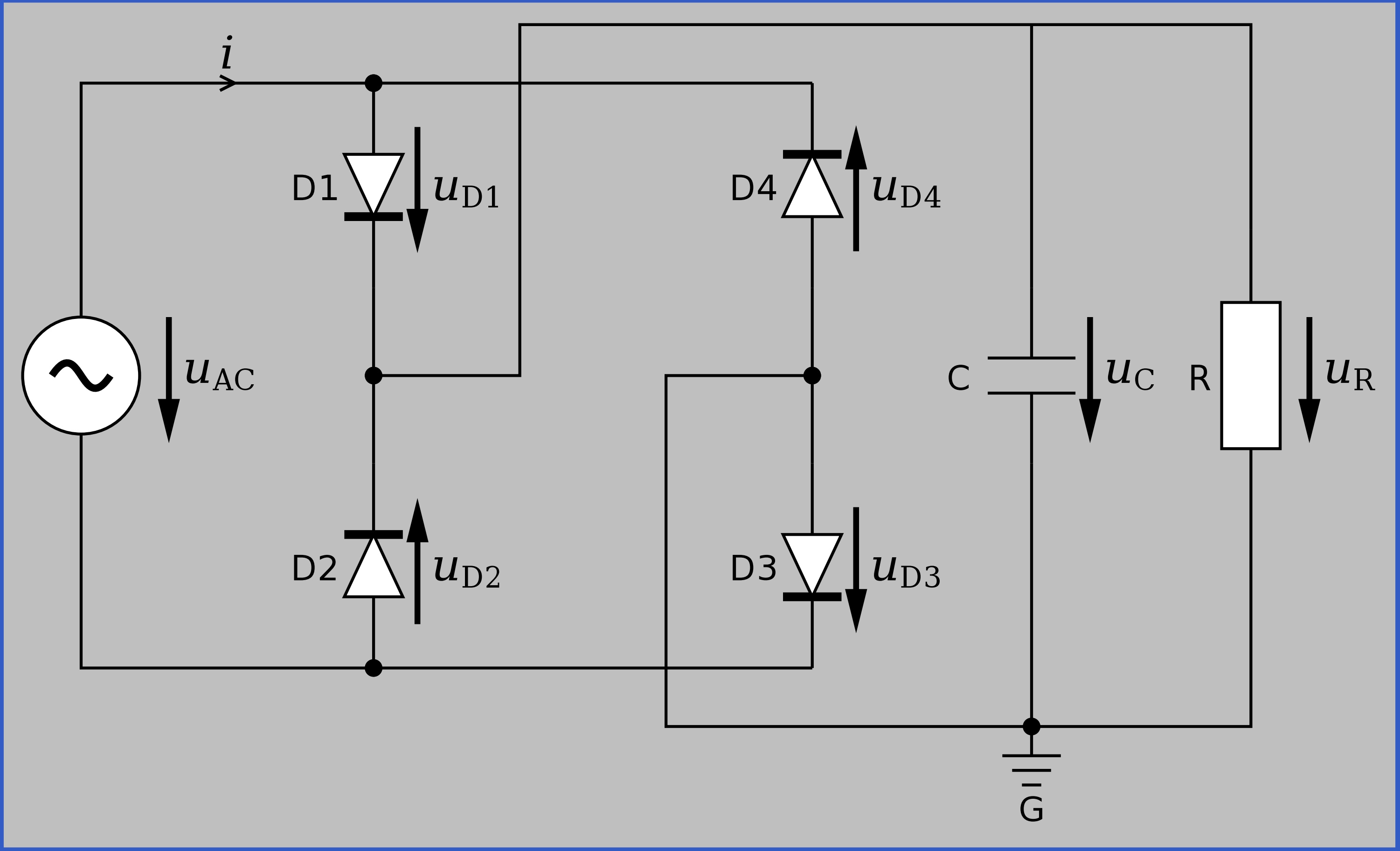
FHM is thus characterised by ***iterative staging***:

1. Compute model ("flat" system of equations)
2. Simulate (solve)
3. Repeat

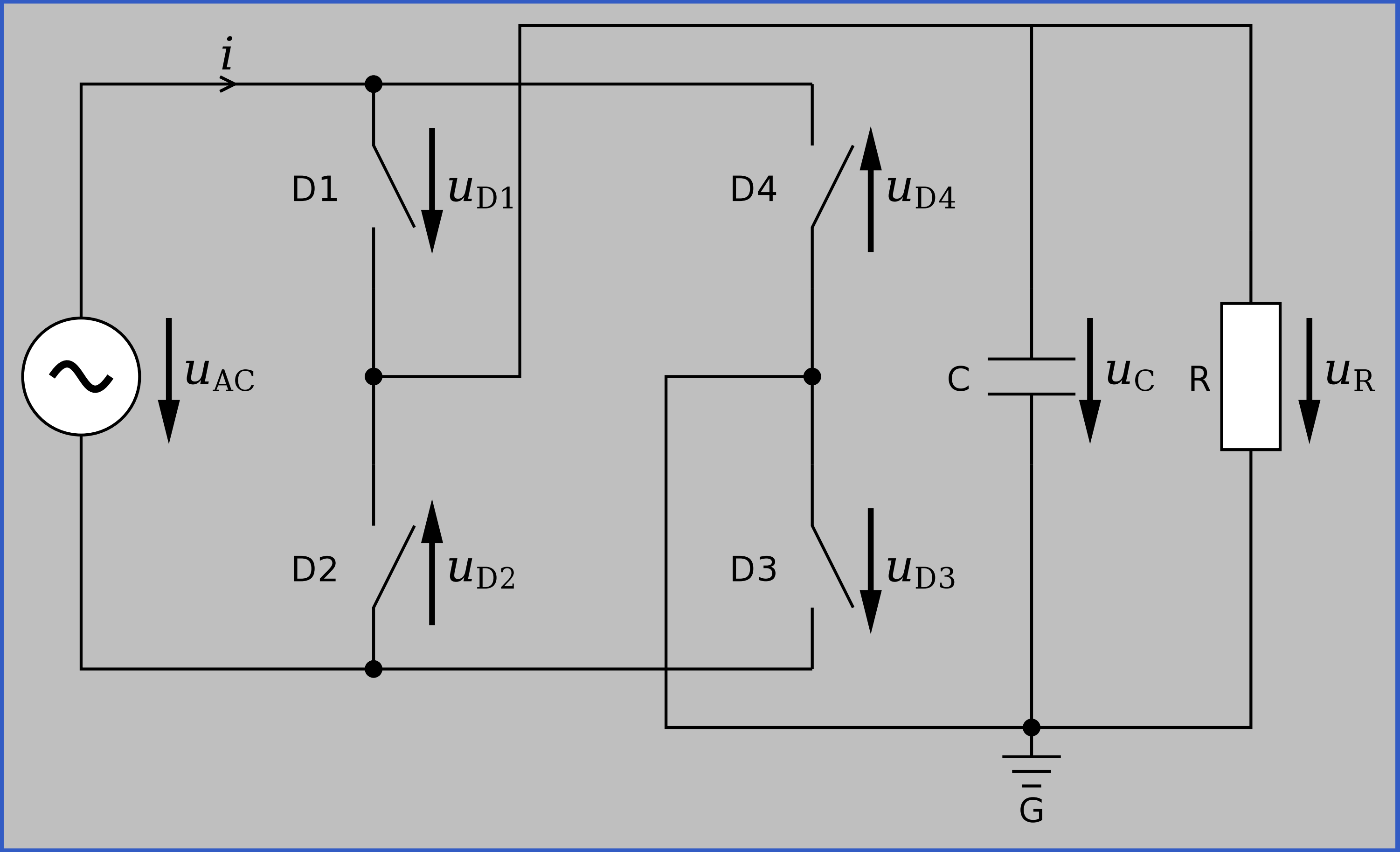
The FHM Setting (5)



Example: Ideal Diodes (1)



Example: Ideal Diodes (2)



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- Can each individual equation system fragment in principle form part of a solvable system?

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- The same holds for:

$$\begin{aligned} x - y + z &= 1 \\ z &= 2 \end{aligned} \tag{2}$$

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- Composing (1) and (2):

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Does have a solution.

- However, the following fragment is over-constrained:

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  - ***Structurally non-singular system***
  - ***Equation-variable balance***

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- Structural non-singularity is neither a necessary nor sufficient condition for solvability.
- However typical solvers are predicated on the system being structurally non-singular.
- Insisting on structural non-singularity thus makes sense and is not overly onerous.

Structural Non-singularity (2)

Structural singularities can be discovered by studying the ***incidence matrix***:

Equations | Incidence Matrix

$$\begin{aligned} f_1(x, y, z) &= 0 \\ f_2(z) &= 0 \\ f_3(z) &= 0 \end{aligned}$$

$$\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \end{array}$$

Structural Non-singularity (3)

Maybe we can annotate signal relation types with incidence matrices?

$$foo :: SR\,(Real, Real, Real) \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

$$\begin{aligned} & foo = \mathbf{sigrel}\ (x_1, x_2, x_3)\ \mathbf{where} \\ & \quad f_1\ x_1\ x_2\ x_3 = 0 \\ & \quad f_2\ x_2\ x_3 \quad\;\; = 0 \end{aligned}$$

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Yes, possible, but rather complicated and we need to approximate anyway.

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$$SR\ (\ldots)\ \boldsymbol{n}$$

Yes, possible. And preservation holds! (Joey)

But very weak assurances:

$$\begin{aligned} f(x, y, z) &= 0 \\ g(z) &= 0 \\ h(z) &= 0 \end{aligned}$$

Structural Well-Formedness (1)

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In *Structural Types for Systems of Equations* we develop a notion of structural well-formedness for modular systems of equations that is:

- a better approximation of structural non-singularity than variable balance;
- less precise, but much more practical (here) than the incidence-matrix-based approach.

Signal relation types are annotated with ***balance***, the number of contributed equations, and types generally carry ***constraints*** on balance variables:

$$(2 \leq m \leq 4, 3 \leq n \leq 5) \Rightarrow SR\,(\ldots)\,m \rightarrow SR\,(\ldots)\,n$$

Structural Well-Formedness (2)

The approach distinguishes between two kinds of variables (number of each kind within parentheses):

- ***interface variables*** (i_Z)
- ***local variables*** (l_Z)

and three kinds of equations:

- ***interface equations*** (i_Q)
- ***mixed equations*** (m_Q)
- ***local equations*** (l_Q)

Total number of equations: $a_Q = i_Q + m_Q + l_Q$.

Structural Well-Formedness (3)

A signal relation is ***structurally well-formed*** (SWF) iff:

1. $l_Q + m_Q \geq l_Z$
2. $l_Q \leq l_Z$
3. $i_Q \leq i_Z$
4. $a_Q - l_Z \leq i_Z$
5. $i_Q \geq 0$, $m_Q \geq 0$, $l_Q \geq 0$

The balance (contribution) of a SWF relation is $n = a_Q - l_Z$.

Structural Dynamism

FHM allows for an evolving system of equations by switching blocks of equations in and out:

$$\textbf{initially}\ [\,;\textbf{when}\ \mathit{condition}_1\,] \Rightarrow$$
$$\quad \mathit{equations}_1$$
$$\textbf{when}\ \mathit{condition}_2 \Rightarrow$$
$$\quad \mathit{equations}_2$$
$$\ldots$$
$$\textbf{when}\ \mathit{condition}_n \Rightarrow$$
$$\quad \mathit{equations}_n$$

What about structural well-formedness?

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Exactly one switch-branch active at any point.
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  Loses too much information.
- ***Fair Approach***: branches are reconcilable.

Structural Dynamism and SWF (2)

A switch-block is ***reconcilable***, contributing i interface equations, m mixed equations, l local equations, iff i, m, l satisfying the following constraints for each branch k can be found:

6. $i \geq i_k \geq 0$
7. $l \geq l_k \geq 0$
8. $m \leq m_k - (i - i_k) - (l - l_k)$
9. $i + m + l = i_k + m_k + l_k$

Note: Interestingly, m may be negative!

Why No Preservation? (1)

Consider:

$$
\begin{aligned}
& \mathit{foo} = \textbf{sigrel}\ (x, y)\ \textbf{where} \\
& \qquad \textbf{local}\ z \\
& \qquad f\ (x, y, z) = 0 \\
& \qquad g\ (x) = 0 \\
& \mathit{fie} = \textbf{sigrel}\ (u)\ \textbf{where} \\
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Both foo and fie are structurally well-formed (why?) with balance 1 and 0, respectively.

Why No Preservation? (2)

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Reduction turned a structurally well-formed relation into one that is ill-formed.

Correctness properties?

The following seems plausible:

> If $t : C \Rightarrow SR\;()\;n$ and $\neg\mathrm{satisfiable}(C)$, then there exists a structural configuration (i.e., a particular choice of switch branches), such that t elaborates to a structurally singular system of equations.

We have not yet attempted to prove this.