Towards a framework for the implementation and verification of translations between argumentation models

(Extended away day version)

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Outline

1. Argumentation theory: a perceived problem
2. An introduction and implementation of argumentation frameworks (Dung)
3. Conclusions and future work

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(Even different notions within the topics)

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- Most models are an instantiation of Dung's model (are translatable to)
- Relatively simple data structures/algorithms (complexity still NP-complete or higher for most problems)
- Not too hard to switch between implementations of AF's because of the very basic data structure (a directed graph)

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- Some recent efforts to optimise the evaluation of AF's (and ASP)
- Existing translations from complex models to Dung, however again a lack of implementations
  - Translations are complex
  - Proofs of correctness are complex (page long proofs)

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Result: a verified way to translate (unimplemented) models to an efficiently implemented model.

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Note that not all models imply a strictly formal structure.

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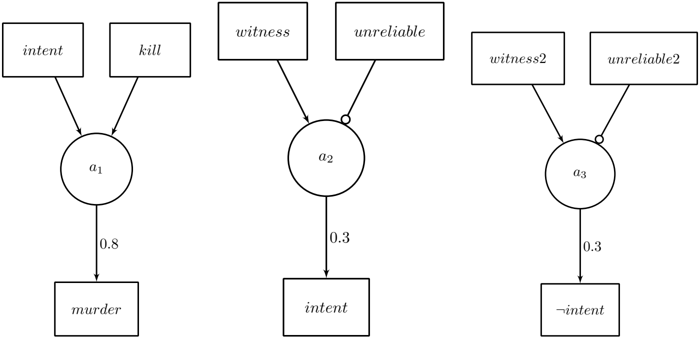
- another inference step called acceptability
- weights, used in acceptability

Carneades argument structures (2)



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For instance: Carneades is translatable to Dung

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And in Haskell:

$$a, b, c :: AbsArg$$
$$a = \texttt{"A"}$$
$$b = \texttt{"B"}$$
$$c = \texttt{"C"}$$

$$AF_1 :: DungAF\ AbsArg$$
$$AF_1 = AF\ [a, b, c]\ [(a, b), (b, c)]$$

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In Haskell:

$$
\begin{aligned}
&setAttacks :: Eq\ arg \Rightarrow DungAF\ arg \rightarrow [arg] \rightarrow \\
&\qquad\qquad\quad arg \rightarrow Bool \\
&setAttacks\ (AF\ \_\ def)\ args\ arg \\
&\quad = or\ [b \equiv arg \mid (a, b) \leftarrow def, a \in args]
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Note that by the required $Eq\ arg \Rightarrow$, Haskell forces us to see that we need an equality on arguments to be able implement these functions.

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&conflictFree :: Eq\ arg \Rightarrow DungAF\ arg \rightarrow [arg] \rightarrow Bool \\
&conflictFree\ (AF\ \_\ def)\ args \\
&\quad = null\ [(a,b) \mid (a,b) \leftarrow def, a \in args, b \in args]
\end{aligned}
$$

Acceptability

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&acceptable :: Eq\ arg \Rightarrow DungAF\ arg \rightarrow arg \rightarrow \\
&\qquad\qquad [arg] \rightarrow Bool \\
&acceptable\ af@(AF\ \_\ def)\ a\ args \\
&\quad = and\ [setAttacks\ af\ args\ b \mid (b, a') \leftarrow def, a \equiv a']
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\begin{array}{l}
f :: Eq\ arg \Rightarrow DungAF\ arg \rightarrow [arg] \rightarrow [arg] \\
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The grounded extension is the minimally acceptable set.

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\begin{aligned}
&groundedF :: Eq\ arg \Rightarrow ([arg] \to [arg]) \to [arg] \\
&groundedF\ f = groundedF'\ f\ [\,] \\
&\quad \textbf{where}\ groundedF'\ f\ args \\
&\qquad\qquad \mid f\ args \equiv args = args \\
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Then as expected:

$$groundedF\ f_{AF_1}$$
$$>[\texttt{"A"},\texttt{"C"}]$$

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- Large parts of Dung's definition have been implemented in Haskell,
- Most of these definitions have been formalised in Agda,
- In previous work we implemented Carneades in Haskell,
- Provided a sketch of how to do a translation from Carneades to Dung in Haskell and which properties one would want to prove.

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Formalisation in Agda, the initial work on the translation and all Haskell code is either discussed or linked to in the paper.

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    - A better understanding of the meaning of some of the complexer argumentation models.

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- Implement and translate(?) my generalisation of the ASPIC^+ argumentation model